Vector Spherical Harmonics for Active magnetic shielding

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The aim of this project is to design, construct and test system for active magnetic field compensation.

This system consists of parts:
- Magnetic field readout and (inter-)extrapolation
- Magnetic field generation

The best beginning for design is to find a basis functions, in which the magnetic field will be described.
Vector Spherical Harmonics

- **Vector Spherical Harmonics** - Complete basis of vector functions in spherical coordinates
- Simplifies calculations with $\nabla$ and $\nabla^2$ operators

\[
\begin{align*}
\vec{\Psi}_{lm}(\theta, \phi) & \equiv r \nabla \vec{Y}_{lm}(\theta, \phi) \\
\vec{Y}_{lm}(\theta, \phi) & \equiv \hat{r} \vec{Y}_{lm}(\theta, \phi) \\
\vec{\Phi}_{lm}(\theta, \phi) & \equiv \hat{r} \times \vec{\Psi}_{lm}(\theta, \phi)
\end{align*}
\]
Definition of a problem

\[ \vec{J} = 0 \implies \nabla \times \vec{B} = 0, \]

\[ \vec{B} = -\nabla \varphi_M, \]

\[ \varphi_M = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \phi_{lm}(r) Y_{lm}(\theta, \phi). \]

Outside our 'test' volume \( \vec{j} \neq 0 \):

\[ \nabla \times \vec{B} = \nabla \times \left( \nabla \times \vec{A} \right) = \frac{4\pi}{c} \vec{j}. \]
Application of VSH to our problem

- Let's apply currents:

\[ \vec{j} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} J_{lm}^{(2)} \Phi_{lm} \]

- They give us the following magnetic field:

\[ \vec{B} = -\nabla \varphi_M = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (l+1)r^{l-1} \left( -l \vec{Y}_{lm} - \vec{\Psi}_{lm} \right) \alpha_{lm}, \]

\[ \varphi_M = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \varphi_{lm}(r) Y_{lm}(\theta, \phi) \]

\[ \varphi_{lm}(r) = (l+1)r^l \alpha_{lm}, \]

\[ \alpha_{lm} = \frac{4\pi}{(2l+1)c} \int_0^{\infty} (r')^{-l+1} J_{lm}^{(2)}(r')dr' \]
1st order coils

\[
\Phi_{10}, \quad \mathbb{R}(\Phi_{11}), \quad \mathbb{I}(\Phi_{11})
\]
Relative difference between desired and achieved field

\[
\text{Relative difference} = \left| \frac{\vec{B}_{\text{vsh}} - \vec{B}_{\text{generated}}}{|\vec{B}_{\text{generated}}|} \right|
\]

10 wires

Relative difference, \( x=0.100000 \)

100 wires

Relative difference, \( x=0.100000 \)
Relative difference in function of number of wires

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Active Magnetic Shielding
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Truncated icosahedron

Properties

- 90 edges of 1 length - $a$
- 30 joints of one type
- Diameter is between $2.478a$ (radius of circumscribed sphere) and $2.2673a$ (distance from center of mass to the hexagonal walls)
- 32 walls - 20 hexagonal and 12 pentagonal
Football coils - 1st order

$\Phi_{10}$  $\Re(\Phi_{11})$  $\Im(\Phi_{11})$
Football coils - relative difference, $l = 1$, $m = 0$, 10 wires

Relative difference in function of $r$
Football coils - relative difference, $l = 1, m = 0, \text{100 wires}$
Work status

What has been done?
- The basis has been chosen for describing magnetic field
- The windings of coils have been found
- Verification is completed
- Configuration with football-like structure is being designed

Plans for future
- Next couple of months - build small ($R < 1\text{m}$) test setup (football structure?)
- Since July - build bigger prototype
Spherical coils - $B_x$, $l = 1$, $m = 0$
Spherical coils - $B_y$, $l = 1$, $m = 0$
Spherical coils - $B_z$, $l = 1, m = 0$
Football coils - $B_x, I = 1, m = 0$
Football coils - $B_y$, $l = 1, m = 0$
Football coils - $B_z$, $l = 1$, $m = 0$
Football coils - relative difference, $l = 1, m = 0$