Feynman Diagrams of the Standard Model

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Outlook

- Introduction to the standard model
  - Basic information
- Feynman diagram
  - Feynman rules
  - Feynman element factors
  - Feynman amplitude
- Examples
The Standard Model

- The Standard Model of particles is a theory concerning the electromagnetic, weak and strong nuclear interactions
  - Collaborative effort of scientists around the world
    - Glashow’s electroweak theory in 1960, Weinberg and Salam effort for Higgs mechanism in 1967
    - Formulated in the 1970s

- Incomplete theory
  - Does not incorporate the full theory of gravitation or predict the accelerating expansion of the universe
  - Does not contain any viable dark matter particle
  - Does not account neutrino oscillations and their non-zero masses
The Standard Model

- The standard model has 61 elementary particles.
- The common material of the present universe is the stable particles, e, u, d.
Gauge Bosons

- Force carriers that mediate the strong, weak and electromagnetic fundamental interactions
  - **Photons**: mediate the electromagnetic force between charged particles
  - **W, Z**: mediate the weak interactions between particles of different flavors (quarks & leptons)
  - **Gluons**: mediate the strong interactions between color charged quarks

- Forces are resulting from matter particles exchanging force mediating particles
  - Feynman diagram calculations are a graphical representation of the perturbation theory approximation, invoke “force mediating particles”
Feynman diagram

- Schematic representation of the behavior of subatomic particles interactions
- Nobel prize-winning American physicist Richard Feynman, 1948
- A Feynman diagram is a representation of quantum field theory processes in terms of particle paths
- Feynman gave a prescription for calculation the transition amplitude or matrix elements from a field theory Lagrangian

\[
\frac{\alpha \rightarrow \beta}{\text{(transition rate)}} = \frac{2\pi}{\hbar} |\beta|V_1|\alpha\rangle|^2 \times \left(\text{density of final quantum states}\right)
\]

\[
\langle \beta|V_1|\alpha\rangle + \text{(higher-order terms)} \quad \rightarrow \quad \langle \beta|S|\alpha\rangle
\]

\[
\langle \beta|S|\alpha\rangle = \delta_{\beta\alpha} - i(2\pi)^4 \delta^4(p_\beta - p_\alpha)M_{\beta\alpha} \prod_{i=\alpha,\beta} \frac{1}{\sqrt{(2\pi)^3 2E_i}}
\]

$|M|^2$ is the Feynman invariant amplitude

- Transition amplitudes (matrix elements) must be summed over indistinguishable initial and final states and different order of perturbation theory
What do we study?

Reactions (A+B→C+D+…)

- Experimental observables: Cross sections, Decay width, scattering angles etc…
- Calculation of $\Gamma$ or $\sigma$ based on Fermi’s Golden rule:

\[
d\Gamma = |M|^2 \frac{1}{2E_1} \left[ \left( \frac{d^3 p_2}{(2\pi)^3 2E_2} \right) \left( \frac{d^3 p_3}{(2\pi)^3 2E_3} \right) \ldots \left( \frac{d^3 p_n}{(2\pi)^3 2E_n} \right) \right] (2\pi)^4 \delta^4 (p_1 - p_2 - p_3 \ldots - p_n)
\]

- Cross sections (1+2→3+4+…+n)

\[
d\sigma = |M|^2 \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \left[ \left( \frac{d^3 p_3}{(2\pi)^3 2E_3} \right) \left( \frac{d^3 p_4}{(2\pi)^3 2E_4} \right) \ldots \left( \frac{d^3 p_n}{(2\pi)^3 2E_n} \right) \right] (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 \ldots - p_n)
\]

- Calculation of observable quantity consists of two steps:
  1. Determination of $|M|^2$ → we use the method of Feynman diagrams
  2. Integration over the Lorentz invariant phase space
Feynman rules

- 3 different types of lines:
  - Incoming lines: extend from the past to a vertex and represents an initial state
  - Outgoing lines: extend from a vertex to the future and represent the final state
  - Internal lines connect 2 vertices (a point where lines connect to another lines is an interaction vertex)

(Incoming and outgoing lines carry an energy, momentum and spin)

- Quantum numbers are conserved in each vertex
  - e.g. electric charge, lepton number, energy, momentum

- Particle going forwards in time, antiparticle backward in time

- Intermediate particles are “virtual” and are called propagators
  - “Virtual” Particles do not conserve E, p
    - for \( \gamma \)’s: \( E^2 - p^2 \neq 0 \)

- At each vertex there is a coupling constant

In all cases only standard model vertices allowed

They are purely symbolic! Horizontal dimension is time but the other dimension DOES NOT represent particle trajectories!
Feynman interactions from the standard model

Because gluons carry color charge, there are three-gluon and four-gluon vertices as well as quark-quark-gluon vertices.
• We construct all possible diagrams with fixed outer particles
  Example: for scattering of 2 scalar particles:

\[ \mathcal{M}(1 + 2 \rightarrow 3 + 4) = \]

\[ + \]

\[ + \]

\[ + \ldots \]

- Since each vertex corresponds to one interaction Lagrangian term in the S matrix, diagrams with loops correspond to higher orders of perturbation theory
- We classify diagrams by the order of the coupling constant (this is just perturbation Theory!!)
- For a given order of the coupling constant there can be many diagrams
- Must add/subtract diagram together to get the total amplitude, total amplitude must reflect the symmetry of the process
  - \( e^+e^- \rightarrow \gamma\gamma \) identical bosons in final state, amplitude symmetric under exchange of \( \gamma_1, \gamma_2 \) : \( M = M_1 + M_2 \)
  - Moller scattering: \( e_{i1}^-e_{i2}^- \rightarrow e_{f1}^-e_{f2}^- \) identical fermions in initial and final state, amplitude anti-symmetric under exchange of \((i1,i2)\) and \((f1,f2)\) : \( M = M_1 - M_2 \)
Feynman diagram element factors

• Associate factors with elements of the Feynman diagram to write down the amplitude
  ➢ The vertex factor (Coupling constant) is just the \( i \) times the interaction term in the momentum space Lagrangian with all fields removed
  ➢ The internal line factor (propagator) is \( i \) times the inverse of kinetic operator (by free equation of motion) in the momentum space
    • Spin 0: scalar field (Higgs, pions, …)
      \[ \frac{i}{p^2 - m^2} \]
    • Spin \( \frac{1}{2} \): Dirac field (electrons, quarks, leptons) scalar propagator multiplies by the polarization sum
      \[ \frac{i}{p^2 - m^2} \sum_{\sigma} u(p, \sigma) \bar{u}(p, \sigma) \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{m^2} \right) \]
    • Spin 1: Vector field
      – Massive (W, Z weak bosons)
      – Massless (photons)
        \[ \frac{-ig^{\mu\nu}}{p^2} \]
• External lines are represented by the appropriate polarization vector or spinor
  e.g. Fermions (ingoing, outgoing) \( u, \bar{u} \); antifermion \( \bar{v}, v \); photon \( \epsilon^\mu, \epsilon^{\mu*} \); scalar 1, 1
Feynman rules to extract $M$

1- Label all incoming/outgoing 4-momenta $p_1, p_2, \ldots, p_n$; Label internal 4-momenta $q_1, q_2, \ldots, q_n$.
2- Write Coupling constant for each vertex
3- Write Propagator factor for each internal line
4- write E/p conservation for each vertex $(2\pi)^4 \delta^4(k_1+k_2+k_3)$; $k$’s are the 4-momenta at the vertex (+/− if incoming/outgoing)
5- Integration over internal momenta: add $1/(2\pi)^4 d^4q$ for each internal line and integrate over all internal momenta
6- Cancel the overall Delta function that is left: $(2\pi)^4 \delta^4(p_1+p_2-p_3-\ldots-p_n)$

What remains is:

$-iM$
First order process

- Simple example: $\Phi^4$-theory
- We have just one scalar field and one vertex
- We will work only to the lowest order

\[ L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{4!} \phi^4 \]

\[ iL_1 = -i \frac{g}{4!} \phi^4 \]

\[ iL_1 \rightarrow -i \frac{g}{4!} \]

\[ \mathcal{L}_{\text{free}} \rightarrow (\partial_\mu \partial^\mu + m^2)\phi = 0 \]

\[ \frac{\partial^\mu}{m \rightarrow -ip^\mu} \]

\[ (p^2 - m^2)\phi = 0 \]

\[ \Rightarrow \phi = \frac{i}{p^2 - m^2} \]

\[ -iM = -i \frac{g}{4!} \]

The tree-level contribution to the scalar-scalar scattering amplitude in this $\Phi^4$-theory
Second order processes in QED

\[ e^+ e^- \rightarrow \mu^+ \mu^- \text{ in QED} \]

- There is only one tree-level diagram

\[
-i \mathcal{M} = [\bar{u}(p_3, \sigma_3)(ie\gamma^\nu)v(p_4, \sigma_4)] \frac{-ig_{\mu\nu}}{(p_1 + p_2)^2} [\bar{v}(p_2, \sigma_2)(ie\gamma^\mu)u(p_1, \sigma_1)]
\]

\[
u_1 \equiv u(p_1, \sigma_1)
\]

\[
\mathcal{M} = \frac{e^2}{(p_1 + p_2)^2} [\bar{u}_3 \gamma_\mu v_4][\bar{v}_2 \gamma^\mu u_1]
\]
Thank you