Active magnetic shielding

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2. Principle of operation of Active Magnetic Shielding

3. How it can be done?
   - Vector Spherical Harmonics
     - Current density functions based on VSH
     - Example 1 - single circular loop
     - Example 2 - Helmholtz coils
   - Other methods

4. Applications of magnetic shielding

5. Summary
Neutron EDM

- Electric Dipole Moment - if nonzero, breaks P and T symmetries
- By SM - $d \approx 10^{-32} \text{ e} \cdot \text{cm}$
- SUSY predicts $10^{-25} \text{ e} \cdot \text{cm} > d > 10^{-28} \text{ e} \cdot \text{cm}$
Frequency of precession: $h\nu = |2\mu_n B \pm 2d_n E|$

Measurement done, by changing relative direction of $\vec{E}$.

Difference between parallel and antiparallel fields: $h\Delta\nu = 4d_n E$
## Systematic errors from measurement

<table>
<thead>
<tr>
<th>No.</th>
<th>Effect</th>
<th>Shift (Ref. [26]) $[10^{-27} \text{ ecm}]$</th>
<th>$\sigma$ (Ref. [26]) $[10^{-27} \text{ ecm}]$</th>
<th>$\sigma$ (Phase II) $[10^{-27} \text{ ecm}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Door cavity dipole</td>
<td>-5.60</td>
<td>2.00</td>
<td>0.10</td>
</tr>
<tr>
<td>2.</td>
<td>Other dipole fields</td>
<td>0.00</td>
<td>6.00</td>
<td>0.40</td>
</tr>
<tr>
<td>3.</td>
<td>Quadrupole difference</td>
<td>-1.30</td>
<td>2.00</td>
<td>0.60</td>
</tr>
<tr>
<td>4.</td>
<td>$\mathbf{v} \times \mathbf{E}$ translational</td>
<td>0.00</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>5.</td>
<td>$\mathbf{v} \times \mathbf{E}$ rotational</td>
<td>0.00</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>6.</td>
<td>Second-order $\mathbf{v} \times \mathbf{E}$</td>
<td>0.00</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>7.</td>
<td>$\nu_{\text{Hg}}$ light shift (geo phase)</td>
<td>3.50</td>
<td>0.80</td>
<td>0.40</td>
</tr>
<tr>
<td>8.</td>
<td>$\nu_{\text{Hg}}$ light shift (direct)</td>
<td>0.00</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>9.</td>
<td><strong>Uncompensated $B$ drift</strong></td>
<td>0.00</td>
<td>2.40</td>
<td>0.90</td>
</tr>
<tr>
<td>10.</td>
<td>Hg atom EDM</td>
<td>-0.40</td>
<td>0.30</td>
<td>0.06</td>
</tr>
<tr>
<td>11.</td>
<td>Electric forces</td>
<td>0.00</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>12.</td>
<td>Leakage currents</td>
<td>0.00</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>13.</td>
<td>ac fields</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td><strong>-3.80</strong></td>
<td><strong>7.19</strong></td>
<td><strong>1.31</strong></td>
</tr>
</tbody>
</table>
Apparatus for nEDM at PSi
How to deal with magnetic field contamination?

- Three square-Helmholtz coil pairs connected to Software calculating currents based on readouts from vector Fluxgate magnetometers
- Supression of magnetic field by 3 times
How does it work?

B=0
How does it work?
How does it work?
How does it work?

Current source

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How to achieve it?

- We can use coils and magnetometers connected in such a way that we are sure that coils compensate field only in position of readout.
- Or we can design coils in such a way that they reduce most efficiently field in whole volume.
Physics standing behind it

Gauss Law
\[ \nabla \cdot \vec{D} = \rho_f \]

Gauss Law for magnetism
\[ \nabla \cdot \vec{B} = 0 \]

Faraday’s Law of induction
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial T} \]

Ampere’s Law
\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]

Assumptions
- \( \rho = 0 \) - We have only conductors in our volume - no polarization
- \( \vec{D}(\vec{x}, t) = \epsilon \vec{E}(\vec{x}, t) \) i \( \vec{B}(\vec{x}, t) = \mu \vec{H}(\vec{x}, t) \) - Linear and isotropic media
Inside our volume there will be $\vec{J} = 0$

It all leads to equations for magnetic field:

$$\nabla \times \vec{B} = 0$$
$$\nabla \cdot \vec{B} = 0$$

We can define:

$$\vec{B}(r) = -\nabla \phi_m$$

Then:

$$\nabla^2 \phi_m = 0$$
We are looking for an orthogonal basis for vector function, which will help us in solving Physical Equation (involving $\nabla$ operator).

Let’s take scalar field:

$$f(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm}(r) Y_{lm}(\theta, \phi)$$

$$Y_{lm} = \frac{1}{N} P_{l}^{m}(\cos \theta) e^{im\phi}$$

We can get vector field by simply taking a gradient of $f$:

$$\nabla f = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( Y_{lm} \nabla f_{lm} + f_{lm} \nabla Y_{lm} \right) =$$

$$= \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( \frac{d}{dr} f_{lm}(r) Y_{lm} \hat{r} + f_{lm} \nabla Y_{lm} \right)$$

---

\[ \nabla f = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (Y_{lm} \nabla f_{lm} + f_{lm} \nabla Y_{lm}) = \]

\[ = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( \frac{d}{dr} f_{lm}(r) Y_{lm} \hat{r} + f_{lm} \nabla Y_{lm} \right) \]

- We can easily see two parts of the Vector Spherical Harmonics:

\[ \vec{\Psi}_{lm}(\theta, \phi) \equiv r \nabla Y_{lm}(\theta, \phi) \]
\[ \vec{Y}_{lm}(\theta, \phi) \equiv \hat{r} Y_{lm}(\theta, \phi) \]

- The third part is \( \vec{\Phi}_{lm}(\theta, \phi) \):

\[ \vec{\Phi}_{lm}(\theta, \phi) \equiv \hat{r} \times \vec{\Psi}_{lm}(\theta, \phi) \]
Vector Spherical Harmonics

This basis is complete - it means that we can write every vector field by following:

\[ \mathbf{V}(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( V_{lm}^r \mathbf{Y}_{lm} + V_{lm}^{(1)} \mathbf{\Psi}_{lm} + V_{lm}^{(2)} \mathbf{\Phi}_{lm} \right) \]

where:

\[ V_{lm}^r = \int d\Omega \, \mathbf{V} \cdot \mathbf{Y}_{lm}^* \]

\[ V_{lm}^{(1)} = \frac{1}{l(l+1)} \int d\Omega \, \mathbf{V} \cdot \mathbf{\Psi}_{lm}^* \]

\[ V_{lm}^{(2)} = \frac{1}{l(l+1)} \int d\Omega \, \mathbf{V} \cdot \mathbf{\Phi}_{lm}^* \]
Applying VSH to our problem

\[ \vec{J} = 0 \implies \nabla \times \vec{B} = 0, \]
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\[ \vec{J} = 0 \implies \nabla \times \vec{B} = 0, \]

\[ \vec{B} = -\nabla \phi_M, \]

\[ \phi_M = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \phi_{lm}(r) Y_{lm}(\theta, \phi). \]
Appling VSH to our problem

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\[ \vec{B} = -\nabla \phi_M, \]

\[ \phi_M = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \phi_{lm}(r) Y_{lm}(\theta, \phi). \]

Let’s put \( \vec{J} \neq 0: \)

\[ \nabla \times \vec{B} = \nabla \times \left( \nabla \times \vec{A} \right) = \frac{4\pi}{c} \vec{J}. \]
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For \( A_{lm}^{(2)} \) (\( \vec{A} = \sum \sum A_{lm}^{(2)} \vec{\Phi} \)) this equation leads to:

\[ \nabla \times \left( \nabla \times A_{lm}^{(2)} \vec{\Phi}_{lm} \right) = \frac{4\pi}{c} J_{lm}^{(2)} \vec{\Phi}_{lm} \]

What brings us to differential equation for \( A_{lm}^{(2)} \):

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} A_{lm}^{(2)} \right) - \frac{l(l + 1)}{r^2} A_{lm}^{(2)} = -\frac{4\pi}{c} J_{lm}^{(2)} \]

Green function (solution for \( J_{lm}^{(2)} = \delta(\vec{r} - \vec{r}') \)):

\[ A_{lm,G}^{(2)} = \frac{4\pi}{(2l + 1) c} \frac{r^L}{r'^{l+1}} (r')^2 \]
\begin{align*}
A_{lm,G}^{(2)} &= \frac{4\pi}{(2l + 1) c} \frac{r_<^l}{r>^{l+1}} (r')^2, \\
(r>, r<) &= \begin{cases}
(r, r') & \text{for } r > r', \\
(r', r) & \text{for } r < r'.
\end{cases}
\end{align*}
\[ A_{lm, G}^{(2)} = \frac{4\pi}{(2l + 1)c} \frac{r_l^l}{r_l^{l+1}} (r')^2, \]

\[(r^+, r^-) = \begin{cases} (r, r') & \text{for } r > r', \\ (r', r) & \text{for } r < r'. \end{cases} \]

\[ A_{lm}^{(2)} = r^l \frac{4\pi}{(2l + 1)c} \int_{0}^{\infty} (r')^{-l+1} J_{lm}^{(2)}(r') \, dr' \equiv r^l \alpha_{lm}, \]
\[ A_{lm, G}^{(2)} = \frac{4\pi}{(2l + 1)c} \frac{r^l}{r} \frac{r^l+1}{(r')^2}, \]

\((r', r)\) for \(r < r'\),

\((r, r')\) for \(r > r'\).

\[ A_{lm}^{(2)} = r^l \frac{4\pi}{(2l + 1)c} \int_0^\infty (r')^{-l+1} J_{lm}^{(2)}(r')dr' \equiv r^l \alpha_{lm}, \]

\[ \vec{B} = \nabla \times \vec{A} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \alpha_{lm} \nabla \times \left( r^l \vec{\Phi}_{lm}(\theta, \phi) \right) \]

\[ = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( -\frac{l(l+1)}{r} r^l \vec{Y}_{lm} - \frac{1}{r} \frac{d}{dr} r^{l+1} \vec{\Psi}_{lm} \right) \alpha_{lm}. \]
\[ \vec{B} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( -l(l+1)r^{l-1}\alpha_{lm} \vec{Y}_{lm} - (l+1)r^{l-1}\alpha_{lm} \vec{\Psi}_{lm} \right), \]

\[ \vec{B} = -\nabla \phi_M = -\sum_{l=0}^{\infty} \sum_{m=-l}^{l} \nabla \left( \phi_{lm}(r) Y_{lm} \right) = \]

\[ = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( -\frac{d}{dr} \phi_{lm}(r) \vec{Y}_{lm} - \frac{\phi_{lm}(r)}{r} \vec{\Psi}_{lm} \right), \]

So:

\[ \phi_{lm}(r) = (l+1)r^l\alpha_{lm} = \frac{4\pi}{c} r^l \frac{l+1}{2l+1} \int_{0}^{\infty} (r')^{-l+1} J^{(2)}_{lm}(r')dr', \]

So, if we have \( \phi_{lm} \) given, we can easily find \( \vec{J} \) on the sphere:

\[ \vec{J} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} J^{(2)}_{lm} \Phi_{lm}. \]
Current density functions based on VSH

\[
\phi_{lm}(r) = (l + 1) r^l \alpha_{lm} = \frac{4\pi}{c} r^l \frac{l + 1}{2l + 1} \int_0^\infty (r')^{-l+1} J_{lm}(r') dr',
\]
Current density functions based on VSH

\[ \phi_{lm}(r) = (l + 1) r^l \alpha_{lm} = \frac{4\pi}{c} r^l \frac{l + 1}{2l + 1} \int_0^\infty (r')^{-l+1} J_{lm}(r')dr', \]

\[ \Phi_{1,0} = -\left( \frac{3}{4\pi} \right)^{1/2} \sin \theta \hat{\phi}, \]

\[ \Phi_{1,1} = -\frac{3}{8\pi} e^{i\phi} \left( \cos \theta \hat{\phi} - i \hat{\theta} \right), \]

\[ \Phi_{2,0} = -3 \left( \frac{5}{4\pi} \right)^{1/2} \sin \theta \cos \theta \hat{\phi}, \]

\[ \Phi_{2,1} = \left( \frac{15}{8\pi} \right)^{1/2} e^{i\phi} \left( (1 - 2 \cos^2 \theta) \hat{\phi} + i \cos \theta \hat{\theta} \right), \]

\[ \Phi_{2,2} = \left( \frac{15}{8\pi} \right)^{1/2} e^{2i\phi} \left( \cos \theta \hat{\phi} - i \hat{\theta} \right). \]
\( \Phi_{1,0} \)
\( \vec{\Phi}_{20} \)
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Circular loop

\[ \vec{J}(r, \theta, \phi) = I \delta(\cos \theta) \frac{\delta(r - R)}{R} \hat{\phi} \]
Results

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Example 2 - Helmholtz coils

\[ \vec{J}(r, \theta, \phi) = I \left( \delta(\theta - \beta) + \delta(\theta - \pi + \beta) \right) \frac{\delta(r - R)}{R} \hat{\phi} \]

\[ \beta = \arctan(2) \]
Results

-1.0 -0.5 0.0 0.5 1.0
y
-1.0
-0.8
-0.6
-0.4
-0.2
-0.0

Bz @TD
VSH l=5
VSH l=3
VSH l=1

Exact

-1.0 -0.5 0.0 0.5 1.0
y

-0.05
-0.04
-0.03
-0.02
-0.01

Difference
VSH l=5
VSH l=3
VSH l=1

2 4 6 8 10 12 14 16 18 20
l
0.3
0.4
0.5
0.6
0.7

rmax

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"Cellural" coils approach

Principle of working

- Large number of small coils
- Each coil with separate current source
- Genetic algorithms to calculate currents
"Cellural" coils approach

Principle of working
- Large number of small coils
- Each coil with separate current source
- Genetic algorithms to calculate currents

Features
- Easier to design mechanically
- More complicated calculation part
- Still needs investigation...
Magnetocardiography

- Measurement of magnetic field generated by currents flowing in human heart
- Usage of SQUIDs
- Comparing to ECG is non-contact, more accurate and possible map creating
- Very small signal, need to use shields
- Other application of magnetic shields for medicine: Magnetoencephalography and Magnetogastrography

Example of diagnosis based on MCG

Example of diagnosis based on MCG


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Active magnetic shielding
Summary

What has been done?

- Found basis functions for magnetic field description
- Found current densities necessary for generating such field

What's next?

- Further investigation, concerning convergence to real data
- Check other solutions (small coils)
- Later, build test setup

Aim is construct final setup for next generation of nEDM experiment, which will have shielding factor of $10^3 - 10^4$ for noise of frequencies $0.01 - 100$ Hz.

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